Landau theory of 2nd order phase transitions

Phase transitions

I order

Latent heat absorbed / released

$$Q = T(S_2 - S_1) = -T \frac{\partial F}{\partial T}\Big|_1^2$$

(Gas - liquid)

II order

$S_1 = S_2$

$\frac{\partial F}{\partial T}$ continuous

($\frac{\partial^2 F}{\partial T^2}$ discontinuous)

Superconductive transition, paramagnet - ferromagnet

// Sometimes people talk about infinite - order phase transitions

---

Focus on <u>2nd order</u> phase transitions

Occur without a rapid transformation of the phase $\longrightarrow$ A continuous phase transition

---

$M \neq 0$	$M = 0$
Ferromagnet	Paramagnet

// Assume a given volume and temperature

$$F = F_0 + \vec{I}\vec{M} + AM^2 + \vec{C}\vec{M}^3 + BM^4 + \ldots$$

$$F = F_0 + \alpha \vec{M} + AM^2 + \vec{C}\vec{M}^3 + BM^4 + \ldots$$

near the transition point.

One should have $\alpha = 0$. (also by symmetry)

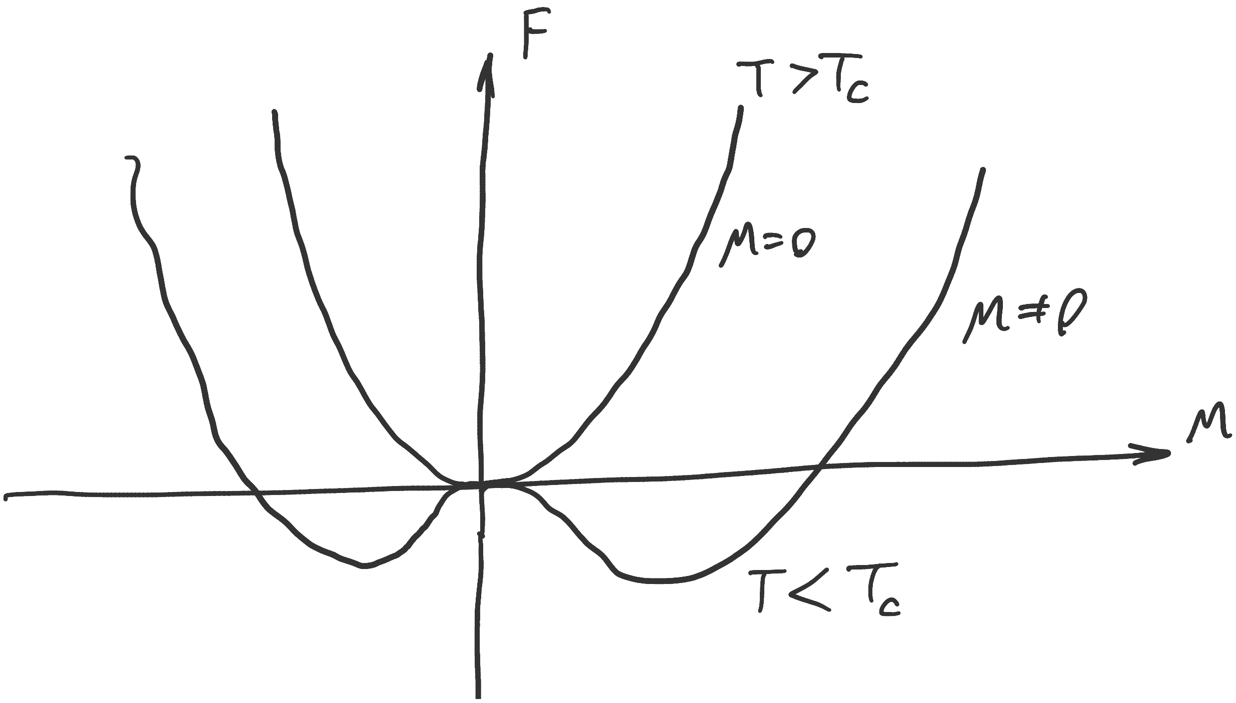
Coeff. $A = A(V, T)$ depends on the volume and the temperature. Otherwise there will always be a local maximum or a minimum across the transition

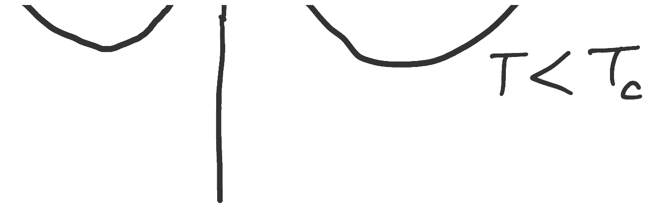
$$F \approx F_0 + AM^2$$

At the transition it changes sign

$$A = a(T - T_c)$$

$$F = F_0 + a(T - T_c)M^2 + BM^4$$





Consider the „ordered" phase with $M=0$
It corresponds to $T<T_c$

$$2BM^2 - a(T_c - T) = 0$$

$$\rightarrow M = \pm\left(\frac{a(T_c-T)}{2B}\right)^{\frac{1}{2}}$$

Note: the magnetisation spontaneously breaks symmetry
Instead of the free energy we would use the thermodynamic potential $\varphi(P,T)$ if the system was under constant pressure P and T.

The entropy

$$S = -\frac{\partial F}{\partial T} = S_0 - aM^2 - a(T-T_c)\frac{\partial M^2}{\partial T}$$

Use $M^2 = \frac{a(T_c-T)}{2B}$ below the transition

$$S = \begin{cases} S_0, & T > T_c \\ S_0 + \frac{a^2(T-T_c)}{B}, & T < T_c \end{cases}$$

$$S = \begin{cases} \\ S_0 + \frac{a^2 (T-T_c)}{2B}, & T < T_c \end{cases}$$

Heat capacity: (near the transition)

$$C = \begin{cases} C_0, & T > T_c \\ C_0 + \frac{a^2 T_c}{2B}, & T < T_c \end{cases}$$

Note: $\frac{\partial^2 F}{\partial T^2}$ is discontinuous

C_p, C_V and a lot of other quantities are discontinuous

Let's apply some external field H

$$F = F_0 + a(T-T_c) M^2 + B M^4 - H M$$

Note: an arbitrarily small field H will lead to a finite magnetisation M

Minimisation:

$$2a M (T-T_c) + 4 B M^3 - H = 0$$

In the limit $H \to 0$ in the disordered phase neglect the $\propto M^3$ term

$$2a M (T-T_c) - H = 0$$

$$2a\ M(T-T_c) - H = 0$$

$$\chi = \frac{M}{H} = \frac{1}{2a(T-T_c)}$$

$$\chi \propto \frac{1}{T-T_c}$$ - Curie-Weiss law

Below the transition

$$\frac{dM}{dH}\left[2a(T-T_c) + 6B\,\frac{a(T_c-T)}{2B}\right] = 1$$

$$\frac{dM}{dH} \cdot a(T_c - T) = 1$$

$$\chi = \frac{dM}{dH} = \frac{1}{a(T_c-T)}$$